

An evaluation of Cyclotron Effective Mass and Electron g-factor in Semiconductor Quantum Well

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ABSTRACT

Using the theoretical formalism of M. De Dios-Leyva *et al.* (2006), we have evaluated the cyclotron effective mass, 2D cyclotron effective mass and g_{11} -factor for GaAs-Ga_{0.65}Al_{0.35}AS quantum well both as a function of magnetic field B (T) and also as a function of well width (Å). Our theoretical evaluated results are in good agreement with the experimental data.

Keywords: Cyclotron effective mass g -factor, Semiconductor quantum well, effective mass approximation, Landau states.

1. INTRODUCTION

The understanding of the physics of wells (QWs), and quantum well wires (QWWs), Quantum dots (QDs) and super lattices (SLs) has been a great interest to solid state physics, several studies have been performed to evaluate the physical properties of these systems¹⁻⁶. The possible use of electron spins in the architecture of solid state based quantum computer has

raised special attention in the study of the behaviour of the electron spin coupled with the external magnetic field. In such type of studies the fundamental importance has been given that no loss can occur when the spins transport information. This goal may be achieved by manipulating the electron g -factor in semiconductor heterostructure and designing appropriate external gate control devices. The cyclotron effective mass and electronic g -factor have possible application

and also in the interpretation of experimental data in this research. These factors play an important role in the magneto-optical and magneto-transport studies optically detected nuclear response experiments, spin electronic and quantum beats measurements. These are also applied the integer quantum Hall Effect.

For the calculation of electron g-factor and cyclotron effective mass one needs the detailed understanding of the interaction between the external applied magnetic field and the electronic state of semiconductor hetrostructure. One new technique like electron spin resonance, spin quantum beats, spin Raman scattering experiments and capacitance and energy spectroscopic. Lattice effects on the orbital contribution, quantum confinement and applications of hydrostatic pressure and external electric/magnetic field may considerably modify conduction electron g-factor both in magnitude and sign in different semiconductor hetrostructure. Theoretical point of view both electron g-factor and cyclotron effective mass provide an excellent tool for testing the band structure electronic calculation in the dimensional semiconductor hetrostructure.

In this paper, we have evaluated cyclotron effective mass (M_c/M_0) and 2D cyclotron effective mass (M_{2D}/M_0) as a function of magnetic field B (T). We have also evaluated (m_c/m_0) and g_{11} factor as a function of well width (A^0) for GaAs and $Ga_{0.65}Al_{0.35}As$ QW keeping ($L=50A^0$). Our theoretical evaluated results are in good agreement with the experimental data.

2. MATHEMATICAL FORMULA USED IN THE EVALUATION

One uses Ogg-McCombe Hamiltonian¹¹⁻¹⁸ within the effective mass approximation and in the forth order of $\vec{k} \cdot \vec{p}$ perturbation theory. This Hamiltonian acts on the two fold Γ_6 spin degenerate conduction band of the bulk materials of the GaAs-Ga_{1-x}Al As quantum well under an applied magnetic field parallel in the growth z-direction. This is written as

$$H = \frac{\hbar^2}{2m^*} \vec{K} \vec{I} + \frac{1}{2} g \mu_B B \sigma_z + \Gamma \sigma \cdot \tau + a_1 \vec{K}^4 \vec{I} + \quad (1)$$

$$\frac{a_2}{l_B^4} \vec{I} + a_3 [\{k_x^2, k_y^2\} + \{k_y^2, k_z^2\}] \vec{I} + a_4 B \sigma_z + a_5 \{\sigma \cdot K, k_z B \vec{I}\} + a_6 B \sigma_z k_z^2 + V(z) \vec{I}$$

Where $\vec{I}$ is the 2x2 unit matrix

$$\hat{K} = \hat{k} + e\hat{A}/\hbar c$$

$$\hat{K} = -i\nabla$$

Landau gauge is used for the vector potential

$$\hat{A} = -yB\hat{x}$$

$a_i = 1-6$ are constants taken from ref. 14 for equal values for the well and barrier materials.

$V(z)$ is square well confining potential taken from $Ga_{1-x}Al_xAs$ and GaAs band gap offset.¹⁹

m^* and g are the z-growth direction position dependent conduction electron effective mass and Landu g-factor.

Γ is constant associated with spin orbit term (due to the fact that GaAs has no. inversion

symmetry), $l_B = \sqrt{\frac{\hbar c}{eB}}$ is the Landau length,

μ_B is the Bohr magneton $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ is the Pauli matrices and $\hat{\tau}$ is a vector operator with components such as $\hat{\tau}_x = \hat{k}_y \hat{k}_x \hat{k}_y - \hat{k}_z \hat{k}_x \hat{k}_z$ and corresponding cyclic permutation. The above Hamiltonian is a 2 x 2 conduction band effective Hamiltonian.¹³⁻¹⁵ This Hamiltonian includes effects of non-parabolicity by taking into account the coupling between the lowest Γ_{6c} conduction band, Γ_{7v} and Γ_{8v} valance bands and the Γ_{7c} and Γ_{8c} p - anti bonding conduction bands.

The 2nd term in RHS of eqn. (1) is the Zeeman contribution, the second order in $\vec{k}$ spin dependent term (with the factor a_4 , a_5 , and a_6) together with the third one is spin orbit interaction). The third order $\vec{k}$ contributes the changes in the hetrostructure effective g-factor. The terms in a_1 and a_3 govern the energy dependence of the Cyclotron effective mass where as term with a_2 gives the diamagnetic shift of the Landau electronic length.

Let the Eigen function of H be

$$\psi(r) = \phi(y, z) \frac{e^{ik_x x}}{\sqrt{L_x}} \quad (2)$$

Here k_x is good quantum number as $\hat{H}$ does not depend on x explicitly. L_x is QW length along x direction. $\psi(\vec{r})$ and $\phi(y, z)$ are two component wave function. Now we write the Hamiltonian^{20,21}

$$\hat{H} = \hat{H}_0 + \hat{W} \quad (3)$$

where $\hat{W}$ is the perturbed Hamiltonian and may be neglected. This only contributes small correction to the

energy levels.²² The Eigen function of $\hat{H}$ (or $\hat{H}_0$) may be written as

$$\psi_+(r) = \frac{1}{\sqrt{L_x}} e^{ik_x x} (\phi_n(y) f_{n,m}^+(z)) \quad (4)$$

and

$$\psi_-(r) = \frac{1}{\sqrt{L_x}} e^{ik_x x} \begin{pmatrix} 0 \\ \phi_n(y) f_{n,m}^-(z) \end{pmatrix} \quad (5)$$

Where the symbols $\pm$ corresponds to ($\uparrow$) spin up and ($\downarrow$) spin down states respectively. Eigen functions $\phi_n(y)$ is 1D harmonic oscillator is given by

$$\phi_n(y) = \frac{1}{[n! 2^n l_B \pi^{1/2}]^{1/2}} e^{-\frac{(y-y_0)^2}{2l_B^2}} H_n\left(\frac{y-y_0}{l_B}\right) \quad (6)$$

Where H_n are the Hermite polynomials, $y_0 = K_x l_B$ is the orbit centre position and $E_n = \hbar \omega_c (n+1/2)$ with $n=0, 1, 2, \dots$ and $\omega_c = eB/m^*c$ are the 1D energies. The sub-index $m = 1, 2, 3, \dots$ in both $f_{n,m}^\pm(z)$ and is associated with the electronic levels $E_{n,m}^\pm$. This indicates $E_{n,m}^\pm$ m-th QW confined energy and $n=0, 1, 2, \dots$ for given m represents the corresponding sub bands energy level.²⁴

The Cyclotron effective mass is given by

$$m_c^{\uparrow\downarrow} = \frac{\hbar e B / c}{[E_{n+1}^{\uparrow\downarrow}(B) - E_n^{\uparrow\downarrow}(B)]} \quad (7)$$

The two-dimensional (2D) Cyclotron effective mass is given as

$$m_{2D}^{\uparrow\downarrow} = \frac{(\hbar e B / c)(n+1/2)}{[E_n^{\uparrow\downarrow}(B) - E_n^{\uparrow\downarrow}(B=0)]} \quad (8)$$

The parallel g-factor is

$$g = \frac{[E_n^{\uparrow}(B) - E_n^{\downarrow}(B)]}{\mu_B B} \quad (9)$$

3. DISCUSSION OF RESULTS

In this paper, using the theoretical formalism developed by M. De. Dios-Leyva *et al.*,²⁵ we have theoretically evaluated electronic Landau level energy, Cyclotron effective mass (M_c/m_0), 2D Cyclotron effective mass (m_{2D}/m_0) and g_{11} -factor for $L=50\text{\AA}$ GaAs-Ga_{0.65}Al_{0.35}As QW as a function of as a function of the growth direction applied magnetic field. The results are shown in table T₁, T₂, T₃ and T₄ respectively. We have also evaluated cyclotron effective mass (m_c/m_0) for GaAs-Ga_{0.65}Al_{0.35}As QW as a function of well width. The theoretical results were compared with experimental data I²⁵ and II²⁶. The results are shown in table T₅. In another evaluation, we have obtained the theoretical result of g_{11} -factor for GaAs-Ga_{0.65}Al_{0.35}As QW as a function of well width ($\text{\AA}$). The results are compared with another two experimental data I²⁷ and II which is shown in table T₆²⁸.

In table T₁, we have shown the electronic levels of the first $m=1$ sub bands of Landau levels. The non-linear behaviour of the $E_{n,m}^{\pm}$ Landau electronic levels is due to the presence of non-parabolic terms in the Hamiltonian. Our theoretical result

shows that the effects of nonlinearity are stronger for the highest Landau levels and also for large values of the applied magnetic field as compared with the effects on the lowest Landau states. The above mentioned properties of the Landau energy levels lead to the behaviour of the Cyclotron masses as shown in table T₂ and T₃. We have evaluated Cyclotron mass (m_c/m_0) for GaAs-Ga_{0.65}Al_{0.35}As QW ($L=50\text{\AA}$) for $n=0, 1, 2, 3$ and 4 keeping $m=1$. Our evaluated results of (m_c/m_0) increases as a function of $B(\text{T})$ for all n . The increase is linear for two values of n and nonlinear for large value of n . Similar behaviour is also observed for 2D cyclotron mass as a function of $B(\text{T})$ shown in table T₃. The first dependent g_{11} -factor is shown in table T₄. In this case also our evaluated results increases as a function of $B(\text{T})$ for all values of n . It appears that field depends on g_{11} factor is due to the magnetic field effect on the electronic Landau states as well as to the confinement or barrier penetration of the electron wave function due to the presence of Ga_{0.65}Al_{0.35}As barrier. We have also calculated Cyclotron effective mass (m_c/m_0) and g_{11} factor both as a function of well width ($\text{\AA}$) for GaAs-Ga_{0.65}Al_{0.35}As QW. The calculated result were compared with the experimental data I²⁶ and II²⁷ for (m_c/m_0) shown in table T₅ and another experimental data I²⁸ and II²⁹ for g factor shown in table T₆. Our evaluate results of (m_c/m_0) is larger in magnitude for experimental data I and small for experimental data II as a function of well width. The evaluated results of (m_c/m_0) decrease as a function of well width as per experimental observation. Similar trend is also obtained for g factor.

Table T₁

An evaluated results of electronic Landau levels as a function of the growth direction applied magnetic field for $L=50\text{\AA}$, GaAs-Ga_{0.65}Al_{0.35}As Quantum Well for $m=1$ sub-bands of Landau states.

Magnetic field B(T)	n=0	n=1	n=2	n=3	n=4	n=5
0	72.3	77.8	86.5	90.5	95.6	98.2
5	74.6	80.5	97.5	98.4	105.4	110.4
10	78.5	85.8	108.5	110.2	119.2	121.8
12	81.2	92.5	112.3	115.8	125.4	132.6
14	83.0	98.8	125.6	130.6	136.8	140.8
15	85.8	106.5	132.3	138.6	144.3	152.6
16	87.2	118.2	139.7	145.3	155.5	162.4
18	89.0	123.6	147.5	154.2	161.7	172.5
20	91.5	135.5	153.6	160.5	172.8	181.6
22	93.2	140.8	159.2	169.8	182.5	190.5
24	95.5	144.6	164.2	175.2	194.3	198.2
25	97.8	155.3	173.6	185.3	199.5	205.3

Table T₂

An evaluated results of Cyclotron effective mass (m_c/m_0) for GaAs-Ga_{0.65}Al_{0.35}As QW as a function of growth direction magnetic field ($m=1$), $L=50\text{\AA}$.

B(T)	←-----(m_c/m_0)-----→ ($m=1$)				
	n=0	n=1	n=2	n=3	n=4
0	0.082	0.084	0.092	0.096	0.104
5	0.084	0.088	0.098	0.102	0.108
10	0.086	0.090	0.104	0.108	0.112
15	0.089	0.094	0.105	0.114	0.115
18	0.090	0.098	0.110	0.118	0.120
20	0.092	0.102	0.114	0.122	0.125
22	0.094	0.106	0.118	0.126	0.130
24	0.096	0.109	0.122	0.130	0.132
25	0.098	0.112	0.126	0.134	0.136
28	0.102	0.114	0.129	0.138	0.140
30	0.104	0.118	0.130	0.142	0.145

Table T₃

An evaluated results of 2D Cyclotron effective mass (m_{2D}/m_0) for GaAs-Ga_{0.65}Al_{0.35}As QW for L=50Å as a function of growth direction magnetic field B(T).

B(T)	←-----(m_{2D}/m_0)-----→ (m=1)				
	n=0	n=1	n=2	n=3	n=4
0	0.080	0.082	0.084	0.089	0.092
5	0.081	0.086	0.088	0.092	0.099
7	0.083	0.089	0.090	0.096	0.106
9	0.085	0.092	0.096	0.099	0.112
10	0.088	0.095	0.099	0.103	0.118
12	0.091	0.099	0.102	0.109	0.122
14	0.094	0.105	0.107	0.112	0.125
15	0.097	0.108	0.115	0.118	0.129
18	0.099	0.112	0.120	0.125	0.135
20	0.102	0.115	0.126	0.130	0.138
22	0.106	0.118	0.129	0.135	0.142
24	0.109	0.120	0.132	0.138	0.143
25	0.112	0.124	0.136	0.142	0.146
28	0.115	0.128	0.139	0.144	0.150
30	0.118	0.130	0.142	0.147	0.152

Table T₄

An evaluated results of g_{11} factor for L=50Å GaAs-Ga_{0.65}Al_{0.35} QW as a function of the growth dimension applied magnetic field.

B(T)	←-----(g_{11} -factor)-----→ (m=1)				
	n=0	n=1	n=2	n=3	n=4
0	0.122	0.124	0.132	0.204	0.217
5	0.124	0.128	0.206	0.247	0.268
7	0.126	0.132	0.258	0.268	0.305
9	0.128	0.136	0.297	0.358	0.362
10	0.130	0.139	0.325	0.392	0.428
12	0.134	0.143	0.367	0.435	0.495
14	0.136	0.146	0.405	0.495	0.573
15	0.138	0.149	0.435	0.558	0.645
17	0.140	0.156	0.468	0.592	0.708
19	0.144	0.159	0.506	0.675	0.786
20	0.148	0.163	0.558	0.706	0.825
22	0.152	0.168	0.605	0.768	0.889
24	0.156	0.172	0.674	0.804	0.922
26	0.159	0.178	0.703	0.855	0.962
28	0.163	0.184	0.768	0.903	1.032
30	0.165	0.190	0.804	0.947	1.057

Table T₅

An evaluated results of Cyclotron effective mass for GaAs-Ga_{0.65}Al_{0.35} QW as a function of well width (Å). The theoretical results were compared with expt. I and expt. II.

Well width (Å)	Cyclotron Effective Mass (m_c/m_0)		
	Theoretical	Expt. I ²⁵	Expt. II ²⁶
5	0.115	0.109	0.118
10	0.108	0.104	0.114
15	0.102	0.100	0.109
20	0.097	0.093	0.107
30	0.092	0.089	0.103
40	0.086	0.085	0.092
50	0.084	0.080	0.087
60	0.080	0.076	0.083
70	0.076	0.072	0.080
80	0.073	0.069	0.076
90	0.070	0.066	0.072
100	0.069	0.062	0.069
110	0.067	0.060	0.065
120	0.065	0.058	0.062
130	0.062	0.056	0.060
150	0.060	0.054	0.058

Table T₆

An evaluated results of g_{11} -factor for GaAs-Ga_{0.65}Al_{0.35}As QW as a function of well width. The theoretical results were compared with the experimental result I and II.

Well width (Å)	g_{11} -factor		
	Theoretical result	Expt. I ²⁷	Expt. II ²⁸
5	0.546	0.584	0.687
10	0.538	0.468	0.562
20	0.512	0.402	0.485
30	0.504	0.352	0.395
40	0.497	0.306	0.324
50	0.482	0.205	0.267
60	0.465	0.186	0.205
70	0.247	0.105	0.125
80	0.134	0.032	0.062
90	0.076	-0.007	-0.004
100	0.065	-0.058	-0.096
120	-0.067	-0.126	-0.146
140	-0.125	-0.148	-0.205
150	-0.246	-0.257	-0.269
160	-0.324	-0.305	-0.325
180	-0.364	-0.348	-0.355
200	-0.405	-0.386	-0.402

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